Why should you care about dependent types?

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Why I care about dependent types

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Type systems Research

- All programs (that do something)
- Programs that do what you want
  - "Equivalent" to a well-typed program, but much easier to write
  - Expressiveness
- Programs that type check
  - Similar to a correct program but the type system can't rule it out
  - Verification
Why Dependent Types?

- **Verification**: Dependent types express application-specific program invariants that are beyond the scope of existing type systems.

- **Expressiveness**: Dependent types enable flexible interfaces, of particular importance to generic programming and metaprogramming.

- **Uniformity**: The same syntax and semantics is used for computations, specifications and proofs.

Program verification is “just programming”
Dependent types and verification

- Haskell prelude function, only defined for non-empty lists:

  ```haskell
  head :: a list -> a
  head (x : xs) = x
  head [] = error "no head"
  ```

- Is "head z" a correct program? Haskell’s type checker can’t tell.
With dependent types

- Datatype that tracks the length of the list at compile time

```haskell
data Nat = O | S Nat

data Vec (a : *) (n : Nat) where
  nil  : Vec a O
  cons : a -> Vec a n -> Vec a (S n)

head :: \(x:\text{Nat}\). Vec a (S x) -> a
head (cons x xs) = x
-- head nil case impossible!
```

- If “head z” typechecks, then z must be non-nil.
Indexed datatypes encode proofs

Inductive is_redblack : tree → color → nat → Prop :=
  | IsRB leaf: ∀c, is_redblack E c 0
  | IsRB_r: ∀tl k tr n, 
    is_redblack tl Red n → Red nodes must have
    is_redblack tr Red n → Black parents
    is_redblack (T Red tl k tr) Black n
  | IsRB_b: ∀c tl k tr n, 
    is_redblack tl Black n → Black nodes can have arbitrary parents
    is_redblack tr Black n →
    is_redblack (T Black tl k tr) c (S n)

Expressiveness

- What about programs that do what you want, but don’t type check?

- Generic programming
  - Types can be calculated by programs
  - Program execution can depend on types

- Embedded Domain-Specific Languages
  - Application-specific type checking
  - Building a programming language is hard!
  - Dependently-typed meta-languages
Generic Programming

• User-defined generic traversals
  – Operations defined over representations of the type structure, in a type-preserving way
  – Eliminates boilerplate code. Aids development & refactoring

• Examples:
  children (BinOp Plus e1 e2) == [e1; e2]
  freshen (If (Var “x”) (Var “y”) (Var “z”)) ==
    (If (Var “x0”) (Var “y0”) (Var “z0”))
  arbitrary / shrink for random test generation
Embedded Domain Specific Languages

• EFFECT, embedded in Idris
  – algebraic model of effects in types
  – alternative to Monad Transformers
  – extensible to new effects

• Bedrock & VST, embedded in Coq
  – low-level safe C-like language for safe systems programming
  – tactics for generating proofs about memory safety

• Ivory, embedded in Haskell
  – low-level safe C-like language for safe systems programming
  – generates C, linked with RTOS and loaded onto quadcopter
What are the research problems in designing dependently-typed languages?
Effective program development

• How can we make it easier to create and work with dependently-typed programs?
  – Specifications and proofs can be long... sometimes longer than the programs themselves

• Research directions:
  – Embedded domain-specific languages
  – Tactics (special purpose language to generate programs)
  – Type/proof inference
  – Theorem provers (SMT solvers, etc.)
  – IDE support: view development as interactive (cf. Ulf Norell ICFP 2013)
  – Incremental development
    • Once you have stated a program property, why not use it for testing first?
Efficient Compilation

• Consider this function:
  
  \[
  \text{safe\_head} : \prod (x: \text{list } a). \text{non\_empty } x \rightarrow a
  \]
  
  \[
  \text{safe\_head} (\text{cons } x \ \text{xs}) \_ = x
  \]

Proof argument

• How do we divide computational arguments from specificalional arguments?
  
  • Idris/Epigram – let the compiler figure it out
  
  • GHC (and many others) – syntactically distinguish them
  
  • Coq – type system sort distinction (Prop / Set)
  
  • Trellys, ICC* (and others) – type system analysis
Non-termination

• Consistency proofs for logic require all programs to terminate
• Programmers don’t
• What to do?
  – Require proofs to be values
  – Nontermination monad (model infinite computation via coinduction)
  – Partial type theories (Nuprl, Zombie)
• Chris Casinghino, Vilhelm Sjöberg, Stephanie Weirich. “Combining Proofs and Programming in a Dependently-Typed Language”, Session 1a tomorrow
Type checking requires deciding type equality.... and types contain programs

When are two programs equal?
- When they are beta equal? $$(\lambda x. x) 3 = 3$$
- (See Richard’s talk on Closed Type Families, Friday)
- When they are beta/eta equal? $$(\lambda x. \text{plus } x \ y) = \text{plus}$$
- When they are both proofs of the same thing?
  - $p1, p2 : A = B$ implies $p1 = p2$
- When their relevant parts are equal?
- Univalence: still more yet...

Many other semantic issues
- Predicativity vs. Impredicativity
- Inductive datatypes & termination
How to get started?
Reading List

• Per Martin Löf. *Constructive mathematics and computer programming*, 1982


• Barendregt. “*Lambda Calculi with Types.*” Handbook of Logic in Computer Science II, 1992

• Harper, Honsell, Plotkin. “*A Framework for Defining Logics.*” *JACM* 1993

• Aspinall and Hoffman. “Dependent types.” *ATTAPL*, 2004

• Sørensen and Urzyczyn, *Lectures on the Curry-Howard Isomorphism*, 2006

• *Homotopy Type Theory: Univalent Foundations of Mathematics*, 2013
Pick a language and play with it

- **Agda**: See wiki for tutorials, watch invited talks from ICFP 2012 (McBride) & 2013 (Norell)
- **Coq**: *Certified Programming with Dependent Types* (Chlipala) *Software Foundations* (Pierce et al.)
- **Idris**: Tutorials and videos at http://www.idris-lang.org/ (Brady)
- **F-star**: Security-focus, compiles to Javascript and F# (Swamy et al.)
- **GHC**: Singletons (Eisenberg & Weirich) and Hasochism (Lindley & McBride)
Implement your own language!

• We are still learning about the role of dependent types in programming
  – There is plenty still to learn by experimenting!

• Don’t have to start from scratch
  – Lectures on implementing Idris (www.idris-lang.org)
  – My OPLSS 2013 lectures & pi-forall github repository